

Analysis of Grade XI Students' Conceptual Understanding of the Law of Sines and Cosines Based on APOS Theory at SMA Negeri 1 Jayapura

¹ Muhammad Sally Sabrilillah, ² Bettisari Napitupulu, ³ Pitriana Tandililing, ⁴ Mayor M.H. Manurung, ⁵ Elieser Kulimbang

^{1) 2) 3) 4) 5)} Fakultas Keguruan dan Ilmu Pendidikan, Universitas Cenderawasih, Jayapura, Indonesia
Email: sally100603@gmail.com

Corresponding Author: sally100603@gmail.com

Article Information	ABSTRACT
Keywords: <i>conceptual understanding, APOS Theory, law of sines and cosines, think-aloud</i>	This study aimed to describe and analyze Grade XI students' conceptual understanding of the topic of the law of sines and cosines based on APOS Theory (Action, Process, Object, Schema) at SMA Negeri 1 Jayapura. A descriptive qualitative approach was used. Eight research participants were purposively selected from 35 students in Class XI-7 based on a written test, ensuring the representation of two students at each APOS stage. Data were collected through a written test and think-aloud interviews conducted at two different times (TA 1 and TA 2), then analyzed using the Miles and Huberman interactive model and verified through time triangulation. The results showed that students' conceptual understanding of the law of sines and cosines developed hierarchically across the four APOS stages. At the Action stage, students could only execute procedures mechanically without detecting errors. At the Process stage, students could explain steps sequentially and reverse procedures but could not yet independently verify their own results. At the Object stage, students consciously integrated triangle properties into their solutions. At the Schema stage, students could connect the law of sines and cosines with other trigonometric concepts and independently evaluate and select more appropriate approaches. Of the 35 students, the majority (77.1%) remained at the Action stage, indicating that instruction still emphasized procedural mastery over the formation of complete conceptual structures.
Article Information	ABSTRAK
Kata Kunci: pemahaman konsep; teori APOS; aturan sinus dan kosinus; <i>Think aloud</i> .	Penelitian ini bertujuan untuk mendeskripsikan dan menganalisis pemahaman konsep siswa berdasarkan teori APOS (Action, Process, Object, Schema) pada materi aturan sinus dan kosinus di Kelas XI SMA Negeri 1 Jayapura. Jenis penelitian yang digunakan adalah deskriptif kualitatif. Subjek penelitian berjumlah 8 orang yang dipilih secara purposive dari 35 siswa Kelas XI-7 berdasarkan hasil tes tertulis, dengan pertimbangan keterwakilan dua siswa pada setiap tahap APOS. Data dikumpulkan melalui tes tertulis dan wawancara <i>Think aloud</i> yang dilaksanakan pada dua waktu berbeda (TA 1 dan TA 2), kemudian dianalisis menggunakan model interaktif Miles dan Huberman serta diverifikasi melalui triangulasi waktu. Hasil penelitian menunjukkan bahwa pemahaman konsep siswa terhadap materi aturan sinus dan kosinus berkembang secara hierarkis sesuai empat tahap APOS. Pada tahap Aksi, siswa hanya mampu menjalankan prosedur secara mekanis tanpa mendeteksi kekeliruan. Pada tahap Proses, siswa mampu menjelaskan langkah secara runtut dan membalik prosedur, namun belum mampu memeriksa kembali kebenaran hasil secara mandiri. Pada tahap Objek, siswa mampu mengintegrasikan sifat-sifat segitiga ke dalam penyelesaian secara sadar. Pada tahap Skema, siswa mampu mengaitkan aturan sinus dan kosinus dengan konsep trigonometri lain serta mengevaluasi dan memilih pendekatan yang lebih tepat secara mandiri. Dari 35 siswa, sebagian besar (77,1%) berada pada tahap Aksi, mengindikasikan bahwa pembelajaran masih menekankan aspek prosedural dibandingkan pembentukan struktur konseptual yang utuh.
Article History	Received : 03/08/2026 Revised : 18/08/2026 Accepted : 21/08/2026

✉ **Corresponding Author:** (1) Muhammad Sally Sabrilillah, (2) Fakultas Keguruan dan Ilmu Pendidikan, (3) Universitas Cenderawasih, Jayapura, Indonesia, (4) Jayapura, Indonesia, (5) Email: sally100603@gmail.com

1. Introduction

Education plays an important role in shaping a nation's civilization. This is affirmed in Article 31, paragraph (1), of the 1945 Constitution of the Republic of Indonesia, which states that every citizen has the right to education. It is also stipulated in Law Number 20 of 2003 concerning the National Education System, which states that national education functions to develop capabilities and shape the character and dignified civilization of the nation in order to advance the intellectual life of the nation. Student-centered education requires a deep understanding of students' learning potential and should be directed toward the comprehensive development of students' personalities by optimizing their full potential (Amaliyah & Rahmat, 2021). Formal education is systematically organized from primary education through higher education, and through this pathway, students acquire a range of foundational knowledge, attitudes, and skills to develop logical, critical, and systematic thinking abilities (Undang-Undang Republik Indonesia Nomor 20 Tahun 2003). As part of formal education, mathematics plays an important role in developing students' logical and analytical thinking skills because it serves as a foundation for various disciplines, including science, economics, engineering, and information technology. The nature of mathematical activity emphasizes thinking processes such as identifying patterns and relationships, testing conjectures, and estimating results, which require students to adapt their existing knowledge to construct new understanding meaningfully (Suryadi, 2012). At the senior high school level, mastery of mathematical concepts is a fundamental necessity, in accordance with Permendikbud No. 12 Tahun 2024 tentang Kurikulum Merdeka, which emphasizes the importance of understanding mathematical facts, concepts, principles, operations, and relationships and applying them flexibly, accurately, and

appropriately in problem-solving. However, students' mathematical conceptual understanding remains relatively low in practice (Hanifah & Abadi, 2018). This condition is closely related to the abstract nature of mathematics, which requires abstract thinking skills to represent various situations within a mathematical framework (Nurhikmayati, 2017). Hill and Ball (as cited in Tianingrum & Sopiany, 2017) emphasized that mathematical conceptual understanding plays an important role not only for students but also for teachers, who require deep conceptual understanding to explain mathematical logic in everyday life.

The need for strong conceptual understanding becomes increasingly evident in the topic of the law of sines and cosines in trigonometry. Difficulties with this topic generally stem from a weak understanding of fundamental trigonometric concepts, such as angle concepts, trigonometric ratios, and the relationships between sides and angles in triangles (Nurfauziah & Sari, 2018). Empirical findings reinforce this issue: Gusmania and Agustyaningrum (2020) explained that students experience difficulty recalling trigonometric concepts when solving problems, and senior high school students' conceptual understanding of the law of sines and cosines is generally not yet optimal, as indicated by their inability to connect concepts with contextual situations and their tendency to memorize formulas without understanding their meaning.

These conditions underscore the need for a systematic analytical framework to examine students' thinking processes in constructing mathematical concepts. One relevant theoretical framework is APOS Theory (*Action, Process, Object, Schema*), introduced by Dubinsky as a development of Piaget's concept of reflective abstraction. According to Dubinsky (as cited in Yuliana & Ratu, 2018), the learning process for a mathematical

concept occurs through the formation of mental actions, their transformation into processes, their abstraction into objects, and their organization into a schema that can be applied in problem solving. Through this four-stage framework, APOS Theory enables the progressive analysis of students' understanding of the law of sines and cosines, from the procedural use of formulas to the ability to relate these concepts to other trigonometric concepts.

Various previous studies support the relevance of APOS Theory. In the topic of linear programming, Nurajijah, Khaerunnisa, and Hadi (2023) found that students' conceptual understanding was hierarchical across levels of mathematical ability. Malinda and Hidayanti (2022) found a similar pattern in absolute-value problems, whereas Windasari, Prasetyowati, and Shodiqin (2020) confirmed that low-achieving students working on geometric sequences tended to stop at the Action stage. Silvia, Rusmana, and Suhendar (2024) also found that high-achieving students were able to meet all indicators of conceptual understanding in trigonometry, whereas low-achieving students met only three of the five indicators. Based on this description, this study aims to describe and analyze students' conceptual understanding based on APOS Theory in the topic of the law of sines and cosines among Grade XI students at SMA Negeri 1 Jayapura, using a combination of a written test *and the* think-aloud method.

2. Methods

This study employed a descriptive qualitative research design aimed at describing and gaining an in-depth understanding of phenomena occurring in the field based on the actual conditions experienced by the research participants. This study was designed to describe students' conceptual understanding based on APOS Theory in the topic of the law of sines and cosines.

Muhammad Sally Sabrilillah et al.: Analysis of Grade XI Students' Conceptual Understanding of the Law of Sines and Cosines Based on APOS Theory at SMA Negeri 1 Jayapura

Research Site and Timeframe

This study was conducted at SMA Negeri 1 Jayapura, located on Jalan Biak, Kelurahan Kota Baru, Kecamatan Abepura, Kota Jayapura, Papua Province. The data collection timeline was as follows: the written test was administered on May 20, 2026; think-aloud interview 1 (TA 1) was conducted on May 22, 2026; and *think-aloud* interview 2 (TA 2) was conducted on May 26, 2026. This timeline served as the basis for applying the time triangulation technique to ensure data validity.

Research Participants

The research participants consisted of 8 students purposively selected from 35 Grade XI-7 students at SMA Negeri 1 Jayapura during the 2025/2026 academic year. The participants were selected based on the written test results, with two students representing each APOS stage (Action, Process, Object, and Schema). To maintain confidentiality, the researchers used name initials when reporting the findings.

Research Instruments

The primary instrument in this study was the researcher. Supporting instruments included (1) a written test in the form of a 120-minute essay test consisting of two items on the law of sines and two items on the law of cosines, developed based on the Action, Process, Object, and Schema indicators; and (2) an interview guide to explore students' thinking processes in depth based on APOS Theory. The conceptual understanding indicators used are presented in Table 1.

Table 1. Indicators of Conceptual Understanding Based on APOS Theory for the Topic of the Law of Sines and Cosines

No.	APOS Stage	Measured Ability
1	Action	Students can identify

		the information provided in problems involving the law of sines and cosines. Students can determine what is being asked in a problem. Students can write the appropriate formula for the law of sines or cosines. Students can use the formula to solve problems procedurally.
2	Process	Students can explain the steps for solving problems involving the law of sines and cosines in a logical sequence. Students can reflect on or review the solution steps they have taken. Students can reverse solution procedures to determine unknown elements of a triangle. Students can apply solution procedures to different problem forms.
3	Object	Students can use the definitions of the law of sines and cosines to solve problems. Students can determine whether to use the law of sines or cosines according to the conditions of a triangle. Students can use the properties of triangles to analyze relationships between sides and angles. Students can explain the concepts of the law of sines and cosines as relationships between the sides and angles of a triangle.

4	Schema	Students can relate the law of sines and cosines to other trigonometric concepts. Students can solve contextual problems involving several concepts simultaneously. Students can comprehensively conclude the relationships among formulas, concepts, and solution procedures.
---	--------	--

Data Collection Techniques

Data were collected through a written test administered once to determine the APOS stages of the research participants, *as well as* think-aloud interviews conducted twice (TA 1 and TA 2) as a time triangulation technique. An essay test was used because it can identify students' conceptual understanding in writing. Interviews were conducted in a structured manner using probing questions to obtain more in-depth information.

Data Analysis Techniques

The analysis technique used was the Miles and Huberman interactive model, which includes (1) data reduction—the selection and simplification of raw data from field notes; (2) data display—the systematic organization of information to enable conclusions to be drawn; and (3) conclusion drawing—the continuous interpretation of the meaning of findings throughout the study (Rijali, 2018).

Data Trustworthiness Techniques

Data trustworthiness was established through the application of time triangulation, namely, data collection at three different time points: the written test, TA 1 two days after the test, and TA 2 several days after TA 1. The consistency of data across these three time points was used to assess the consistency of students' conceptual understanding based on APOS Theory, thereby strengthening

confidence that the data obtained were accurate and accountable.

Research Procedures

This study was conducted in four stages: (1) the pre-field stage, including the preparation and processing of permits and the development of research instruments; (2) the fieldwork stage, involving data collection through written tests and *think-aloud* interviews; (3) the data analysis stage, involving the processing, selection, and organization of all data using the planned procedures; and (4) the reporting stage, involving the preparation of a written report in an academic format as a form of scientific accountability.

3. Results and Discussion

This study aims to describe and analyze the conceptual understanding of Grade XI students at SMA Negeri 1 Jayapura based on APOS Theory in the topic of the law of sines and cosines. Data were collected through a written test, think-aloud *interview 1* (TA 1), and *think-aloud* interview 2 (TA 2), which were triangulated over time to ensure data trustworthiness.

Results of APOS Stage Classification in Grade XI/7

A written test consisting of three essay questions on the topic of the law of sines and cosines was administered to 35 Grade XI/7 students to comprehensively map their APOS stage attainment. The scoring results based on the conceptual understanding indicators at each APOS stage are presented in Table 2.

Table 2. Classification of Grade XI/7 Students Based on APOS Stages

No.	APOS Stage	Number of Students	Percentage
1	Action	27	77,1%
2	Process	3	8,6%
3	Object	2	5,7%
4	Schema	3	8,6%
Total		35	100%

1	Action	27	77,1%
2	Process	3	8,6%
3	Object	2	5,7%
4	Schema	3	8,6%
Total		35	100%

Table 2 shows that most Grade XI/7 students, namely, 27 of 35 students (77.1%), reached only the Action stage in their conceptual understanding of the law of sines and cosines. This distribution pattern is consistent with the conditions reported by Gusmania and Agustyaningrum (2020), who found that conceptual understanding of trigonometry was generally still dominated by procedural ability and that few students had attained complete conceptual mastery.

Selected Research Participants

Based on the grouping results in Table 2, the researchers purposively selected eight students, with two students representing each APOS stage. An additional consideration was their ability to communicate their thoughts orally, given that data were also collected through think-aloud interviews. The list of selected research participants is presented in Table 3.

Table 3. List of Selected Research Participants

No.	Participant Code	Gender	APOS Stage (Initial Test)
1	NW	Female	Action
2	MR	Male	Action
3	NK	Female	Process

4	NF	Female	Process
5	ND	Female	Object
6	RA	Female	Object
7	JF	Female	Schema
8	RN	Female	Schema

The eight participants subsequently took part in two *think-aloud* sessions at different times as part of a time triangulation strategy to examine the consistency of the conceptual understanding demonstrated on the written test.

Summary of APOS Stage Attainment Based on Time Triangulation

After the written test, TA 1, and TA 2 results for each participant were analyzed and compared, a valid summary of APOS stage attainment was obtained, as presented in Table 4.

Table 4. Summary of the Eight Participants' APOS Stage Attainment Based on Time Triangulation for Questions 1, 2, and 3

N o.	Participa nt	Questi on 1	Questio n 2	Questi on 3
1	NW	Action	Did not respond	Action
2	MR	Action	Indication of Object (Not Yet Consistent)	Action
3	NK	Process	Action (limited data)	Process
4	NF	Process	Indication of Object	Process

			(Not Yet Consistent)	
5	ND	Object	Schema	Schema
6	RA	Object	Schema	Schema
7	RN	Schema	Did not complete the task	Schema
8	JF	Schema	Schema	Schema

Table 4 shows three important patterns. First, APOS stage attainment was not uniform within individual research participants across different problems, depending on the context and the degree of novelty in the problem structure. Second, Problem 2, which required identifying the properties of an $\sin 120^\circ$ isosceles triangle and using the value in the second quadrant, proved to be a clear differentiator among research participants, as NW and RN did not complete the problem, whereas JF reached the Schema stage. Third, JF consistently reached the Schema stage on all three problems and demonstrated the most stable understanding, whereas NW and MR consistently remained at the Action stage.

Students' Conceptual Understanding at the Action Stage

The Action stage was characterized by the procedural use of formulas without adequate conceptual understanding. Students at this stage were able to identify the given information and substitute values into the formulas for the law of sines and cosines; however, they were unable to check their answers, consistently explain their reasons for selecting a formula, or independently solve problems requiring subsequent steps.

These findings are consistent with Dubinsky and McDonald's (2001) definition of the Action stage, which states that, at this stage,

individuals carry out procedures based on memorized rules without deep understanding and depend on clear instructions or procedural steps. This is also supported by Mulyono (2011), who noted that the characteristics of Action include directly using formulas, applying existing algorithms, following previous examples, requiring detailed steps, and proceeding procedurally.

This tendency for students to remain at a low level of understanding without reaching higher-order reasoning was also found in a similar context by Tandililing et al. (2025), who examined geometry instruction among elementary school students in Papua. Their study showed that most students remained at the visual or procedural level of thinking without progressing to formal reasoning. The same condition was evident at this Action stage, indicating that the predominance of procedural understanding is a consistent pattern in mathematics instruction, particularly for topics requiring relational understanding.

These findings are also consistent with those of Windasari et al. (2020) on the topic of geometric sequences, who found that low-achieving students at the Action stage were only able to distinguish examples from nonexamples and directly use formulas without conceptual flexibility. Similarly, Nurajijah et al. (2023) found that low-achieving students could only reach the Action stage and could not progress to the Process, Object, or Schema stages. This pattern indicates that limitations at the Action stage are not an isolated phenomenon in a single topic or particular context but rather a common challenge in mathematics instruction.

Furthermore, these findings reinforce the view of Gusmania and Agustyaningrum (2020) that students experience difficulty recalling trigonometric concepts and connecting the

concepts of the law of sines and cosines to contextual situations and tend to memorize formulas without understanding their underlying meaning. The inability to detect errors and check answers is a clear manifestation of this condition: once the memorized procedure has been carried out, no internal mechanism prompts students to verify whether the solution is correct and complete.

Students' Conceptual Understanding at the Process Stage

The Process stage showed significant development compared with the Action stage. Students at this stage were able to explain the solution steps sequentially, reverse procedures to determine unknown elements of a triangle, and explain their reasons for using the law of sines based on the conditions of the problem. However, they were not yet able to independently check the answers they obtained, so errors in the final calculation stage went undetected.

These characteristics are consistent with Dubinsky and McDonald's (2001) definition of the Process stage, which states that a process is formed when an action is repeated and reflected upon until it becomes an internal mental construction. At this stage, individuals are able to imagine and explain transformations without having to carry out each step explicitly, including mentally reversing or combining them. The ability to reverse the procedure for the law of cosines provides clear evidence of the internalization of action into a mental process.

These findings are consistent with those of Wahyuningsih and Nissa (2019), who found that several participants at the Process stage were able to follow solution steps systematically but experienced difficulties verifying and proving their answers. Similarly, Malinda and Hidayanti (2022) found that

students with moderate ability showed good development in process ability but remained limited at the Object stage when confronted with problems requiring the comprehensive integration of concepts.

The emergence of initial characteristics of the Object stage is consistent with the view that the APOS hierarchy is gradual rather than discrete. As proposed by Dubinsky & McDonald (2001), transitions between stages occur gradually through the accumulation of reflection and learning experiences. The identification of alternative solution methods indicates that the internalization process is developing toward encapsulation, although it was not yet stable and consistent across the three data sources.

Students' Conceptual Understanding at the Object Stage

At the Object stage, students demonstrated deeper conceptual understanding than students at the Action and Process stages. Students were not only able to carry out procedures sequentially but were also able to consciously use the definitions and properties of triangles to analyze relationships between sides and angles and determine the appropriate law based on the given triangle conditions.

These findings are consistent with Dubinsky & McDonald's (2001) definition of the Object stage, which states that an object is formed when an individual recognizes a process as a complete whole that can be treated as an entity subject to further operations or transformations. The ability to spontaneously infer that a triangle is isosceles and use this inference to determine an angle provides clear evidence of the encapsulation of a process into a cognitive object.

The finding that conceptual understanding may develop ahead of procedural calculation fluency is consistent

with Napitupulu et al.'s (2025) view that students' conceptual understanding of abstract geometry topics can be improved through active engagement and concrete visualization, enabling students to integrate the properties of geometric figures into problem solving more meaningfully. In the context of the law of sines and cosines, the integration of the properties of an isosceles triangle indicates that understanding geometric properties serves as a cognitive bridge that enables students to operate with trigonometric concepts at a higher level.

Significant development toward the Schema stage reinforces the findings of Nurajijah et al. (2023) that students with moderate ability can reach the Object stage and show early indications of Schema ability. This condition is also consistent with the findings of Malinda & Hidayanti (2022), who stated that students in the moderate-ability category may demonstrate good conceptual development even though they are not yet able to integrate all schema components consistently.

Students' Conceptual Understanding at the Schema Stage

At the Schema stage, students demonstrated the highest degree of conceptual flexibility, with the ability to connect the law of sines and cosines with other trigonometric concepts, such as trigonometric identities, related-angle formulas, and trigonometric values in different quadrants. Students at this stage were able to evaluate the validity of the results obtained and independently select the most appropriate solution strategy.

These characteristics are consistent with Dubinsky & McDonald's (2001) definition of the Schema stage, which states that a schema is an organized collection of Actions, Processes, Objects, and other Schemas

interconnected through general principles, thereby forming a flexible framework for solving mathematical problems. The ability to use different but mutually consistent methods and to spontaneously use angle-sum and related-angle formulas constitutes a concrete manifestation of a well-organized schema.

These findings are consistent with those of Wahyuningsih & Nissa (2019), who found that participants who reached the Schema stage were able to formulate and solve mathematical models completely using various approaches. Similarly, Nurajijah et al. (2023) found that high-ability students who reached the Schema stage demonstrated the ability to connect various concepts and independently evaluate solution strategies. Windasari et al. (2020) also identified a similar pattern in the topic of geometric sequences, in which students at the Schema stage were able to solve word problems using various procedures and select the most efficient method.

The most notable finding of the analysis is that the Schema stage is not a static endpoint. The significant consolidation and deepening of schemas in TA 1 and TA 2 indicate that the process of thematization continues through reflection and articulation. The verbalization process through think-aloud was shown to reflect understanding at a deeper level, producing explanations that were more explicit and comprehensive than those presented in the written test. This is consistent with the view that mathematical articulation and communication skills reflect the depth of students' conceptual understanding (Nurfauziah & Sari, 2018).

4. Conclusion

Based on the findings and discussion concerning the analysis of conceptual

understanding based on APOS Theory in the topic of the law of sines and cosines among Grade XI students at SMA Negeri 1 Jayapura, it can be concluded that the conceptual understanding of Grade XI students at SMA Negeri 1 Jayapura regarding the law of sines and cosines was still dominated by the Action stage (77.14%), followed by the Process stage (8.57%), the Object stage (5.71%), and the Schema stage (8.57%). This condition indicates that students' conceptual understanding was still strongly dominated by the Action stage. This dominance reflects that instruction on the law of sines and cosines in Grade XI-7 at SMA Negeri 1 Jayapura still placed greater emphasis on procedural mastery than on conceptual understanding. Most students were not yet able to reflect on the correctness of their results, explain the rationale for selecting formulas, or connect the law of sines and cosines with broader trigonometric concepts. This condition indicates the need to design instruction that explicitly promotes the development of students' conceptual understanding from the Action stage to the Process, Object, and Schema stages, particularly by strengthening their ability to reflect, verify answers, master trigonometric values in different quadrants, and make connections among concepts.

References

- [1].Amaliyah, A., & Rahmat, A. (2021). Pengembangan potensi diri peserta didik melalui proses pendidikan. *At-Ta'Dib*, 5(1), 28–45. <https://doi.org/10.32832/at-tadib.v5i1.19598>
- [2].Dubinsky, E., & McDonald, M. A. (2001). APOS: A constructivist theory of learning in undergraduate mathematics education research. *The Teaching and Learning of Mathematics at University Level*, 275–282. https://doi.org/10.1007/0-306-47231-7_25
- [3].Gusmania, Y., & Agustyaningrum, N. (2020).

- Analisis pemahaman konsep matematis mahasiswa pada mata kuliah trigonometri. *Jurnal Gantang*, 2, 123–132. <https://doi.org/https://doi.org/10.31629/jg.v5i2.2493>
- [4].Hanifah, H., & Abadi, A. P. (2018). Analisis pemahaman konsep matematika mahasiswa dalam menyelesaikan soal teori grup. *Journal of Medives: Journal of Mathematics Education IKIP Veteran Semarang*, 2(2), 235–244. <https://doi.org/10.31331/medives.v2i2.626>
- [5].Malinda, P., & Hidayanti, M. (2022). Analisis kemampuan konseptual peserta didik dalam menyelesaikan soal nilai mutlak berdasarkan teori APOS. *Jurnal Cendekia: Jurnal Pendidikan Matematika*, 6(2), 1482–1492. <https://doi.org/10.31004/cendekia.v6i2.1396>
- [6].Malinda, T. T., & Hidayanti, W. S. (2022). Analisis kemampuan konseptual peserta didik dalam menyelesaikan soal nilai mutlak berdasarkan teori APOS. *Pi: Mathematics Education Journal*, 5(1), 17–26. <https://doi.org/https://doi.org/10.21067/pmej.v5i1.6417>
- [7].Mulyono. (2011). *Teori apos dan implementasinya dalam pembelajaran. I.*
- [8].Napitupulu, B., Ruamba, M., Sally, M., Nur, W. O. C., & Karubaba, S. (2025). Penerapan model pembelajaran kooperatif tipe jigsaw berbantuan alat peraga takebo pada materi bangun ruang. *Science and Technology: Jurnal Pengabdian Masyarakat*, 2(4), 476–482.
- [9].Nurajijah, M., Khaerunnisa, E., & Hadi FS, C. A. (2023). Kemampuan pemahaman konsep matematis siswa berdasarkan teori apos pada materi program linear. *Jurnal Educatio FKIP UNMA*, 9(2), 785–797. <https://doi.org/10.31949/educatio.v9i2.4800>
- [10].Nurfauziah, P., & Sari, V. T. A. (2018). Penerapan bahan ajar trigonometri dengan model matematika Knisley untuk meningkatkan kemampuan berpikir kritis matematik mahasiswa. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, 7(3), 356–362. <https://doi.org/10.24127/ajpm.v7i3.1551>
- [11].Nurhikmayati, I. (2017). Kesulitan berpikir abstrak matematika siswa dalam pembelajaran problem posing berkelompok. *Kalamatika: Jurnal Pendidikan Matematika*, 2(2), 159–176. <https://doi.org/10.22236/kalamatika.vol2n02.2017pp159-176>
- [12].Rijali, A. (2018). Analisis data kualitatif. *Alhadharah: Jurnal Ilmu Dakwah*, 17(33), 81–95. <https://doi.org/10.18592/alhadharah.v17i3.3.2374>
- [13].Silvia, M., Rusmana, I. M., & Suhendar, E. (2024). Analisa pemahaman konsep matematika siswa pada materi trigonometri kelas X SMK Asisi. *Bilangan: Jurnal Ilmiah Matematika, Kebumian Dan Angkasa*, 2(5). <https://doi.org/10.62383/bilangan.v2i5.285>
- [14].Suryadi, D. (2012). *Membangun budaya baru dalam berpikir matematika*. Bandung: Rizqi Press.
- [15].Tandililing, P., Sirampun, E., Kho, R., & Ruamba, M. Y. (2025). Geometric thinking levels in learning quadrilaterals: A Van Hiele-based case study in a Papuan junior high school. *Edumatica: Jurnal Pendidikan Matematika*, 15(2), 124–136. <https://doi.org/10.22437/edumatica.v15i2.43719>
- [16].Tianingrum, R., & Sopiany, H. N. (2017). Analisis kemampuan pemahaman matematis siswa. 440–446.
- [17].Wahyuningsih, H., & Nissa, I. C. (2019). Analisis kemampuan siswa dalam memahami konsep sistem persamaan linier tiga variabel (SPLTV) berdasarkan teori APOS. *Jurnal Ilmiah Mandala Education*, 5(2), 164–171.
- [18].Wahyuningsih, H., Nissa, I. C., & Yuntawati, Y. (2019). Analisis kemampuan siswa dalam memahami konsep sistem persamaan linier tiga variabel (SPLTV) berdasarkan teori APOS siswa kelas X IPS 1 MA Tarbiyatul Mustafid Batu Rimpang. *Media Pendidikan Matematika*, 7(1), 36–50. <https://doi.org/10.33394/mpm.v7i1.1556>

- [19].Windasari, I. Y., Prasetyowati, D., & Shodiqin, A. (2020). Analisis pemahaman konsep berdasarkan teori APOS pada materi barisan geometri di kelas XI SMA Negeri 1 Godong. *MATHEdunesa: Jurnal Ilmiah Pendidikan Matematika*, 417–427.
- [20]., R., Masriyah, & Ekawati, R. (2020). Analisis pemahaman konsep berdasarkan teori APOS pada materi barisan geometri di kelas XI SMA Negeri 1 Godong. *MATHEdunesa: Jurnal Ilmiah Pendidikan Matematika*, 9(2), 338–346.
- [21].Yuliana, D., & Ratu, N. (2018). *Deskripsi kemampuan pemahaman konsep eksponen berbasis teori apos pada siswa sma theresiana salatiga*. 5(1), 51–65.