

Analysis of the Understanding of Straight-Line Equation Concepts Based on Action, Process, Object, Schema Theory

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ABSTRACT

Understanding the concept of straight-line equations is essential as a prerequisite for advanced mathematics. Observations at SMP Negeri 5 Jayapura indicated that eighth-grade students still encountered difficulties, particularly in determining the gradients of parallel and perpendicular lines. This study aimed to describe students' conceptual understanding profiles of straight-line equations based on APOS Theory (Action, Process, Object, and Schema) across high-, medium-, and low-ability categories. The research employed a descriptive qualitative method. Subjects were purposively selected from 27 students based on their MID scores, categorized as high (≥ 90), medium (80–89), and low (< 80), with one student representing each category. The instruments consisted of two essay-type test questions incorporating Papuan cultural contexts and a semi-structured interview guide. The findings revealed that the high-ability student achieved all four APOS stages, the medium-ability student achieved the Action, Process, and Object stages, whereas the low-ability student achieved only the Action stage. Although performance across the Action to Object stages varied, the collective failure of all three subjects in graphical representation indicated cognitive obstacles during the encapsulation process at the Schema stage. These findings recommend the implementation of the APOS-based ACE cycle and special attention to graphical aspects in learning straight-line equations.

Informasi Artikel

Kata Kunci:

APOS;
Garis lurus;
Pemahaman
konsep.

ABSTRAK

Pemahaman konsep persamaan garis lurus penting sebagai prasyarat materi matematika lanjutan. Observasi di SMP Negeri 5 Jayapura menunjukkan peserta didik kelas VIII masih mengalami kesulitan, terutama pada gradien garis sejajar dan tegak lurus. Penelitian ini bertujuan mendeskripsikan profil pemahaman konsep persamaan garis lurus peserta didik berdasarkan Teori APOS (Aksi, Proses, Objek, Skema) pada kategori kemampuan tinggi, sedang, dan rendah. Metode penelitian adalah kualitatif deskriptif. Subjek dipilih secara purposive dari 27 peserta didik berdasarkan nilai MID dengan kategori tinggi (≥ 90), sedang (80–89), dan rendah (< 80), masing-masing satu orang per kategori. Instrumen berupa tes tertulis dua soal uraian dengan konteks budaya Papua dan pedoman wawancara semi-terstruktur. Hasil penelitian menunjukkan peserta didik kemampuan tinggi mencapai keempat tahap APOS, peserta didik kemampuan sedang mencapai tahap Aksi, Proses, dan Objek, sedangkan peserta didik kemampuan rendah hanya mencapai tahap Aksi. Meskipun performa pada tahap Aksi hingga Objek bervariasi, kegagalan kolektif ketiga subjek dalam representasi grafis mengindikasikan adanya hambatan kognitif pada tahap enkapsulasi Skema. Temuan ini merekomendasikan penerapan siklus ACE berbasis APOS dan perhatian khusus pada aspek grafis dalam pembelajaran persamaan garis lurus.

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1. Introduction

Conceptual understanding is a fundamental competency in mathematics learning. Principles and Standards for School Mathematics [1] views conceptual understanding as a cognitive foundation that enables students to reason systematically and evaluate the validity of mathematical arguments rather than merely memorize procedures. Mathematical understanding refers to the ability to absorb material, recall formulas and concepts, estimate the validity of a statement, and apply formulas and theorems to solve mathematical problems [2]. Low levels of conceptual understanding are reflected in students' understanding of straight-line equations.

This topic is taught in Grade VIII and serves as a prerequisite for advanced topics such as linear functions, linear programming, and systems of linear equations in two variables. Mastery of the concept includes slopes, forms of linear equations, procedures for determining the equation of a line through one or two points, and relationships between lines (parallel, perpendicular, and intersecting). The importance of this topic is not consistent with conditions in the field, as students' learning outcomes on this topic remain below average, indicating that their mathematical problem-solving ability is still low when solving straight-line equation problems [3]. Previous studies have revealed that students' conceptual understanding of this topic remains low [3], [4], [5], [6], [7], [8].

A similar phenomenon was identified at SMP Negeri 5 Jayapura, where preliminary observations and interviews with teachers indicated a significant discrepancy in eighth-grade students' mastery of the concept of slope. Students experienced difficulty determining the slope, particularly when a line equation was not presented in the standard form $y = mx + c$. The most dominant difficulty occurred in the topic of relationships between lines, especially in determining the slope of a line perpendicular or parallel to another line whose equation was already known. This indication strengthens the assumption that students' conceptual understanding remains mechanical (based on memorization).

A step-by-step analysis of low conceptual understanding requires an appropriate theoretical framework, namely APOS Theory. According to [9] mathematical mastery is not defined as memorizing facts but as the ability to construct a cognitive structure gradually—from a concrete action, internalized into a mental process, encapsulated into a cognitive object, and finally organized into an interconnected schema. This perspective on cognitive construction is clarified by [10], who define mathematical conceptual understanding as students' ability to explain, interpret, discover, apply, restate concepts, and present concepts in various forms of mathematical representation. [11] also emphasize that mathematical conceptual understanding is important so that students can

connect given problems with previously acquired knowledge. [12], in a study at SMP Negeri 1 Palembang, found that students' conceptual understanding through the APOS stages tended to be good. The ability to restate concepts was the most dominant indicator, whereas applying concepts to problem solving was the least frequent.

At the Action stage, students still depend on step-by-step instructions originating externally—for example, they may substitute a value into $y = 2x + 3$ without being able to explain the reason behind the procedure. When the procedure has been recorded and can be carried out mentally without external stimulation, students enter the Process stage. The Object stage is achieved when slope is no longer merely a calculated number but an entity that can be compared, for example, to determine parallelism ($m_1 = m_2$) or perpendicularity ($m_1 \times m_2 = -1$). The Schema stage, as the highest level, is characterized by the integration of these concepts into a unified network of understanding.

APOS Theory has also been applied by [13] to systems of linear equations in two variables (SPLDV), who found that high- and medium-ability students fulfilled all four APOS indicators, whereas low-ability students fulfilled only the Action and Process stages. This finding reinforces that APOS can distinguish students' levels of understanding.

[12] described students' understanding of straight-line equations through APOS, but did not analyze differences in achievement at each stage according to ability category. [13] distinguished APOS achievement by ability category, but their study focused on SPLDV rather than straight-line equations. Thus, for straight-line equations, there has been no study mapping achievement at each APOS stage according to students' ability categories. The novelty of this study lies in mapping achievement at each APOS stage (Action, Process, Object, and Schema) across three ability categories (high, medium, and low) in the context of straight-line equations. Research on mathematical conceptual understanding of this topic has examined other perspectives, such as gender [2], learning styles [14], and learning independence [11], but has not mapped APOS achievement according to ability category. Therefore, this study focuses on the mapping of APOS stages across three ability levels in learning straight-line equations.

Based on this background, this study aims to describe the profile of eighth-grade students' conceptual understanding of straight-line equations at SMP Negeri 5 Jayapura across high-, medium-, and low-ability categories based on APOS Theory. Theoretically, this study is expected to expand the taxonomy of conceptual understanding of straight-line equations from an APOS perspective and provide empirical contributions to the development of a curriculum based on local contexts.

2. Methods

This study employed a descriptive qualitative design to explore students' mental constructions in depth based on the APOS framework, rather than to test relationships among variables or generalize findings to a broader population. This approach was selected because the study did not aim to statistically measure relationships among variables but to explore the meaning underlying students' thinking as it naturally occurred, with the researcher serving as the primary data-collection instrument [15]. Another characteristic of qualitative research, as emphasized by [16], is its open nature: research questions and data-collection procedures may be adjusted as the fieldwork progresses because the main focus is understanding participants' perspectives rather than testing rigid hypotheses. [17], in a study at SMP Negeri 1 Bandongan, also used a descriptive qualitative approach to identify students' mathematical conceptual understanding of straight-line equations.

The study was conducted at SMP Negeri 5 Jayapura during the second semester of the 2025/2026 academic year. The site was selected based on communication with a mathematics teacher indicating that most eighth-grade students experienced difficulties understanding straight-line equations, particularly the concepts of slopes of parallel and perpendicular lines. The population consisted of all 29 eighth-grade students at SMP Negeri 5 Jayapura. Participants were selected using purposive sampling. Purposive sampling was used rather than random sampling because the objective was not to generalize findings to a broader population but to identify participants who genuinely represented each ability category [15]. [16] add that qualitative researchers intentionally select participants or sites that can help them understand the problem and research questions. Participant selection was conducted in three stages. First, a written test consisting of four items measuring the four APOS stages (Action, Process, Object, and Schema) was administered to all eighth-grade students, with a completion time of 60 minutes. Second, students were grouped into three categories based on their test scores. Third, one student from each category was selected for interview, resulting in three interview participants.

Table 1. Criteria for students' ability on MID scores

Kategori	Rentang Nilai MID	Keterangan
High	≥ 90	Student With MID scores in the high category
Medium	80 – 89	Student With MID scores in the medium category
Low	< 80	Student With MID scores in the low category

category

Based on the criteria in Table 1, three research subjects were obtained: the high-ability subject (ST) had an MID score of 95, the medium-ability subject (SS) had an MID score of 84, and the low-ability subject (SR) had an MID score of 78.

The instruments used in this study consisted of two types: a written test and an interview guide. The two instruments were used to obtain complementary data: the written test captured students' written thinking processes, while the interview guide explored reasoning that could not be captured from answer sheets alone. [18] add that in qualitative research, the researcher serves as the key instrument for collecting data through interviews, observations, and documents.

The written test consisted of two essay questions, each containing four subquestions (eight items in total), developed according to the four stages of APOS Theory. The two questions were presented as word problems incorporating Papuan cultural contexts, namely noken sales and papeda sales at a traditional event. This was intended to make the problems easier for students to understand because the contexts were familiar to their daily lives in Jayapura. The use of contextual problems is consistent with [3], who provided non-routine problems related to everyday life because students were more accustomed to problems modeled by teachers. The maximum total score was 100

Table 2. Blueprint of the written test instrument based on test score

No	Apos Stage	Indikator	Bentuk Soal	Skor
1a	Action	Determine the increase in cost from a table	Essay	8
1b	Process	Complete the table for 4,5, and 6	Essay	10
1c	Object	Construct a straight-line equation	Essay	12
1d	Schema	Determine the cost of 12 noken, determine the number of noken for a given cost, explain the slope and constant, and draw a graph	Essay	20
2a	Action	Determine the increase in cost from a table	Essay	8
2b	Process	Complete the table for 4,5, and 6 portions	Essay	10
2c	Object	Construct a straight-line equation	Essay	12
2d	Schema	Determine the cost of 15 portions, determine the number of noken for a given cost, explain the slope and constant, and draw a graph	Essay	20
Total maximum score				100

As shown in Table 2, Question 1 measured the Action (8 points), Process (10 points), Object (12 points), and Schema

(20 points) stages. Question 2 had the same structure but used a different context. The total maximum score was 100. The test instrument was validated by two expert validators, namely mathematics education lecturers with expertise in straight-line equations, APOS Theory, and qualitative research methodology. The validated aspects included the alignment of the questions with the indicators, wording clarity, and readability. No empirical trial was conducted because of time and access limitations.

The interview guide was semi-structured and consisted of 38 questions developed according to the four APOS stages, with 20 questions for Question 1 (noken) and 18 questions for Question 2 (papada). [18] explain that an interview protocol should be prepared before the interview and should include basic information, an introduction, core questions with probes, and closing instructions. [15] adds that interviews are used as a data-collection technique when researchers seek deeper information from a small number of respondents.

The interview guide was organized into four stages. The Action stage explored students' understanding of how to determine increases in cost from a table and the meaning of the obtained values. The Process stage explored students' ability to complete the table, identify patterns, and determine costs for larger quantities. The Object stage explored students' ability to formulate a straight-line equation from tabulated data and understand the meaning of coefficients and constants. The Schema stage explored students' ability to use equations to determine unknown values, explain the meaning of the slope and constant in the problem context, connect tables, equations, and graphs, and analyze changes that occur when a variable is changed.

Secara rinci, untuk soal 1 (noken), For Question 1 (noken), interview questions 1a measured the Action stage, questions 1b the Process stage, questions 1c the Object stage, and questions 1d the Schema stage. For Question 2 (papada), questions 2a measured the Action stage, questions 2b the Process stage, questions 2c the Object stage, and questions 2d the Schema stage. The interview guide was also validated by the same two expert validators. The interviews were adjusted to students' abilities: the high-ability student was asked all questions, whereas the medium- and low-ability students were questioned only up to the limits of their abilities.

Data collection was conducted using three methods: written tests, interviews, and documentation. [15] states that data collection is the most strategic step in research because the main objective of research is to obtain data. The written test was administered offline (face-to-face) in the classroom for 60 minutes, with all eighth-grade students (27 students) completing two essay questions (eight subitems) and being asked to write their calculation steps completely. Interviews were conducted individually in a quiet room, with the duration adjusted to students' abilities. The interviewer recorded

summaries of students' responses rather than verbatim transcripts, and interviews were stopped when students were no longer able to answer further questions. [18] emphasize the importance of probes such as "Tell me more" or "Could you explain your response more?" to obtain deeper information. Documentation consisted of students' answer sheets, interview summary notes, and photographs of the research activities as evidence of implementation. [15] states that documentation serves as supporting evidence in the form of records of events that have occurred—in this study, answer sheets, interview notes, and photographs.

Data were analyzed through three interrelated stages following the framework of Miles and Huberman as cited by [15]: First, raw data were reduced and focused on information relevant to the research questions. Second, the organized data were presented in tables and narrative descriptions so that patterns could be identified. Third, tentative conclusions were drawn from the emerging patterns and continuously rechecked as new data appeared. [18] similarly describe qualitative data analysis as organizing and preparing data, reading all data, coding, identifying themes, developing interpretations, and presenting data. Data reduction was conducted by scoring students' answer sheets using the rubric, grouping students into three categories (high, medium, and low) based on their MID scores, selecting one subject from each category for interview, and summarizing the interview results in notes. In this study, data reduction means filtering students' answers and interview transcripts so that only portions directly addressing the research questions were retained, making patterns of achievement at each APOS stage easier to identify. Data were presented in tables and narratives. Conclusions were drawn by comparing APOS-stage achievement across the high, medium, and low categories, mapping the APOS stages achieved by each category, and verifying the data to ensure the accuracy of the conclusions. Sugiyono (2013) explains that the third step in qualitative data analysis is conclusion drawing and verification; preliminary conclusions remain tentative and may change when sufficient supporting evidence is not found, whereas conclusions supported by valid and consistent evidence are considered credible. [15] explains that the third step in qualitative data analysis is conclusion drawing and verification; preliminary conclusions remain tentative and may change when sufficient supporting evidence is not found, whereas conclusions supported by valid and consistent evidence are considered credible.

Data trustworthiness was examined using method triangulation and member checking. To maintain confidence in the findings, the study considered four common aspects of qualitative trustworthiness: credibility, transferability, dependability, and confirmability [15]. [18] add that qualitative validity involves checking the accuracy of findings through procedures such as triangulation, member checking,

and clarification of researcher bias. Method triangulation was conducted by comparing data obtained from two different sources, namely written tests and interviews. When the written-test and interview results were consistent, the data were considered valid. When differences occurred, the researcher conducted further investigation, such as reexamining the answer sheets or repeating interview questions. [15] explains triangulation as a data-collection technique that combines various data-collection techniques and sources. In addition to method triangulation, the researcher conducted member checking by asking subjects to confirm the accuracy of the summarized interview results. [18] state that member checking involves returning final findings or specific descriptions to participants to determine whether they consider the findings accurate.

3. Results and Discussion

3.1 Results

The study was conducted at SMP Negeri 5 Jayapura with a population of 29 eighth-grade students. Of these, 27 students participated in the written test, while two were absent. Based on the criteria established in the Methodology (Table 1), students were grouped into three ability categories. The distribution of students in each category is presented in Table 3.

Table 3. Distribution of students by ability category

Category	MID Score Range	Number of Students	Percentage
High	≥ 90	4	14,8%
Medium	80 – 89	19	70,4%
Low	< 80	4	14,8%
Total		27	100%

One subject from each category was selected for analysis of the answer sheets. The profiles of the three subjects are presented in Table 4.

Table 4. Research subject profiles

Code	MID Score	Category
ST	95	High
SS	84	Medium
SR	78	Low

Description of Student's Answer Sheets

The answer sheets of the three subjects were analyzed based on the indicators in the instrument blueprint (Table 2). Subject ST (MID score 95) was able to solve all questions correctly and systematically. ST was able to write complete calculation steps, determine increases in cost, complete the tables, formulate straight-line equations, and determine unknown values. However, ST experienced difficulty drawing the

graph, placing the X- and Y-axes in reversed positions. Subject SS (MID score 84) was able to solve most of the questions, but several steps were incomplete. SS was able to determine increases in cost, complete the tables, and formulate straight-line equations. However, SS experienced difficulty drawing the graph and was less able to explain the meaning of the constant. Subject SR (MID score 78) was able to solve only the simplest tasks. SR was able to determine the increase in cost from two data points in the table but could not complete the table, formulate a straight-line equation, or draw the graph.

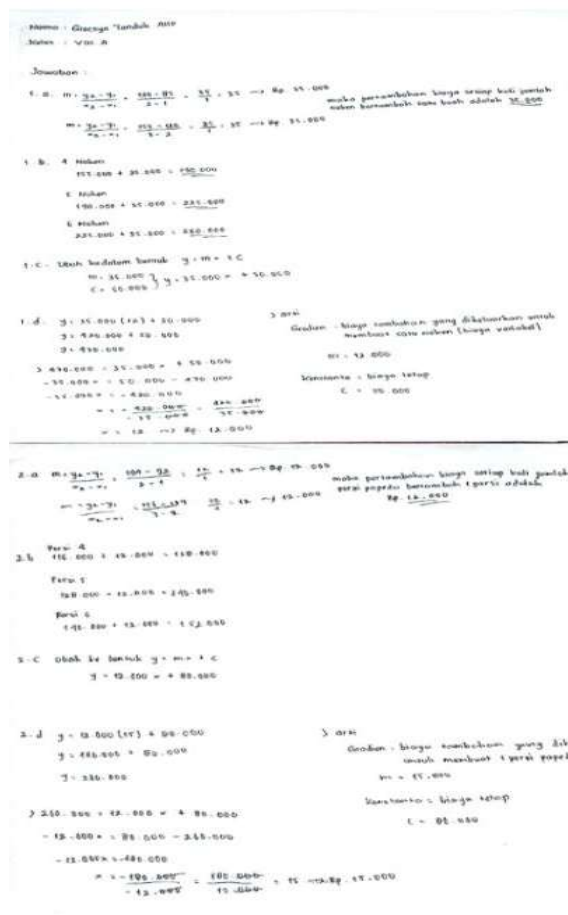


Figure 1. Answer sheet of subject ST.

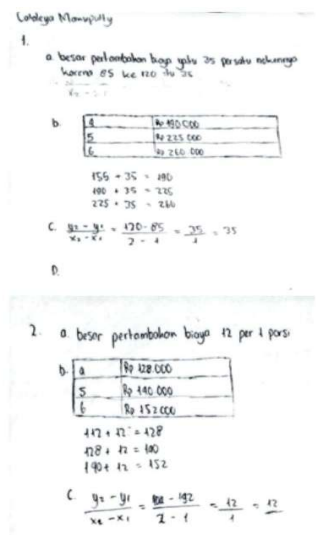


Figure 2. Answer sheet of subject SS.

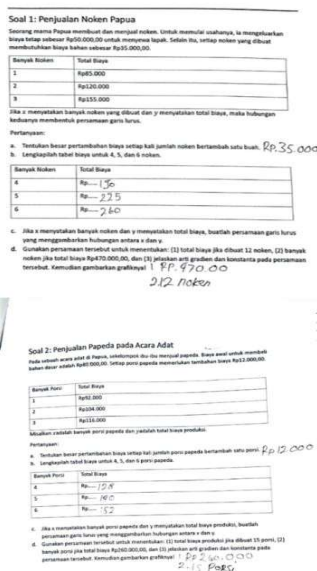


Figure 3. Answer sheet of subject SR.

Table 5. Scores achieved by each (maximum score 100)

Subjek	Skor	Kategori
ST	94	High
SS	60	Medium
SR	16	Low

Table 5 shows that ST obtained a score of 94 (high category), SS obtained a score of 60 (medium category), and SR obtained a score of 16 (low category). The substantial difference in scores among the three subjects indicates a considerable difference in conceptual understanding.

Interview Results

Wawancara Interviews were conducted to explore students' understanding more deeply at each APOS stage. ST was able to answer the interview questions fluently and confidently. ST could determine increases in cost from the table, understand the emerging pattern, formulate a straight-line equation, and explain the meaning of the slope and constant. ST could also use the equation to determine unknown values. However, ST experienced difficulty drawing the graph. SS was able to answer questions at the Action and Process stages but began to show uncertainty at the Object and Schema stages. SS could determine increases in cost and complete the table and formulate a straight-line equation. However, SS could not draw the graph correctly and was less able to explain the meaning of the constant. SR was able to answer only the simplest Action-stage question, namely determining the increase in cost from the table. SR could not complete the table, formulate a straight-line equation, or draw the graph. The interview was stopped because SR was no longer able to answer further questions. Overall, all three subjects were unable to draw the graph of a straight-line equation correctly.

Profile of understanding based on the APOS stages.

Profile of understanding based on the APOS stage

Based on the written-test and interview results, the profiles of the three subjects at each APOS stage are presented in Table 6

Table 6. Students' conceptual understanding profiles based on APOS stages

APOS stage	Indicator	ST	SS	SR
Action	Determine the increase in cost from a table	Achieved	Achieved	Achieved
Process	Complete the table	Achieved	Achieved	Not achieved
Object	Construct a straight-line equation	Achieved	Achieved	Not achieved
Schema	Apply the equation, explain its meaning, and draw a graph	Achieved	Not achieved	Not achieved

Table 6 shows that ST achieved all four APOS stages. SS achieved the Action, Process, and Object stages but had not reached the Schema stage. SR achieved only the Action stage, to a limited extent, and had not reached the Process, Object, or Schema stages.



Figure 4. Atmosphere of students working on a written test.

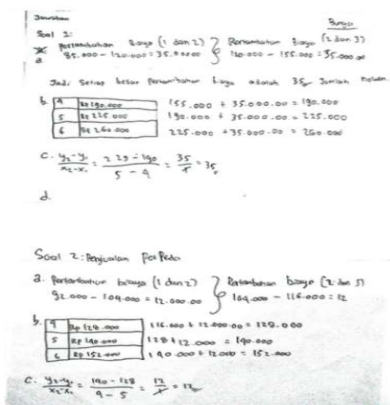


Figure 5. Example of a student's answer sheet.

3.2 Discussion

Understanding of High-Ability Student

The high-ability student (ST) achieved all four APOS stages: Action, Process, Object, and Schema. ST was able to determine increases in cost from the table, complete the table, formulate a straight-line equation, and use the equation to determine unknown values. ST was also able to explain the meaning of slope and constant in the context of the problem. This finding is consistent with [19], who stated that high-ability students can fulfill all indicators of conceptual understanding, including restating concepts, providing examples and non-examples, classifying objects, using procedures, and applying concepts. This finding is also consistent with [20], who reported that students with high conceptual understanding can achieve all APOS stages (Action, Process, Object, and Schema) when solving mathematical problems. [21] similarly found that high-mathematics-ability students were able to achieve all stages of APOS Theory.

Understanding of Medium-Ability Student

Unlike ST, the medium-ability student (SS) achieved the Action, Process, and Object stages but had not reached the Schema stage. SS was able to determine increases in cost from the table, complete the table, and formulate a straight-line equation. However, SS experienced difficulty drawing the graph, reversing the positions of the X- and Y-axes, and was less able to explain the meaning of the constant in the context

of the problem. [11] found a similar pattern, in which students with moderate conceptual understanding still made errors in solving problems and had not yet fulfilled all indicators of conceptual understanding. The result is consistent with [20], which found that medium-ability students were able to solve problems only at the Action, Process, and Object stages, and with [22], who found that errors at the Object stage reached 30%, indicating that students had not fully understood problem-solving procedures. The main difficulty experienced by SS occurred during encapsulation from Object to Schema, in which SS was unable to integrate different representations (table, equation, and graph) into a unified understanding.

Understanding of Low-Ability Student

Meanwhile, the low-ability student (SR) reached only the Action stage in its simplest form. SR was able to determine the increase in cost from the table (Questions 1a and 2a), but was unable to complete the table, formulate a straight-line equation, or understand slope as an object that could be compared. This finding is consistent with [23], who showed that low-ability students had not understood problems well and could not solve questions according to the required procedures. [19] also found that low-ability students were unable to describe problems completely. SR had not yet carried out interiorization (from Action to Process), so the actions performed remained mechanical and dependent on examples. This is consistent with [21], who found that students with low mathematical ability could reach only the Action stage in APOS Theory. Restianingsih et al. (2023) also found that subjects with low self-confidence could solve problems only at the Action stage and frequently made errors. [25] added that students' errors in solving straight-line equation problems were caused by limited broad understanding of the material, lack of accuracy, and low interest in mathematics learning.

Comparison Across Ability Categories

[26] described students' understanding of straight-line equations through APOS in general without distinguishing ability categories. This study complements Agustina et al. by showing that achievement at each APOS stage varies according to students' ability categories. Table 6 clearly shows that high-ability students achieved all APOS stages, medium-ability students achieved the Action, Process, and Object stages, whereas low-ability students achieved only the Action stage to a limited extent. This finding strengthens [27], who identified a similar pattern in SPLDV, confirming that APOS Theory can be used to distinguish levels of students' understanding based on their ability levels in general, not merely in one particular mathematical topic.

A similar finding was reported by Ismail and Manurung in their study at SMP YPPK Teruna Mulia, Jayapura City, on SPLDV using Polya's steps. The study categorized students' problem-solving abilities into three levels: level three (high), level two (medium), and level one (low). Level-three students were able to understand problems, plan solutions, implement the plans, and recheck all answers. Level-two students were able to understand the given problems but were not fully able to plan and implement solutions or recheck their answers. Meanwhile, level-one students were unable to understand certain problems, did not plan or implement solutions, and did not recheck their answers [28]. Thus, the consistency of the

finding that higher student ability is associated with better problem-solving achievement can be observed through both APOS and Polya frameworks, in both straight-line equations and SPLDV.

Learning Implication

Based on the findings that the high-ability student had reached the Schema stage but still experienced difficulty drawing graphs, the medium-ability student was hindered during encapsulation from Object to Schema, and the low-ability student reached only the Action stage, practical implications can be formulated through the implementation of the APOS-based ACE cycle (Activities, Classroom Discussion, Exercises), adapted to the specific difficulties encountered in learning straight-line equations. The ACE cycle is an instructional framework developed to implement APOS Theory in classrooms [29].

During the Activities stage, teachers provide concrete activities to build the Action and Process stages. [30] found that low-ability students experienced difficulty applying basic concepts and therefore required repeated manipulative activities. For low-ability students who have reached only the Action stage, teachers should provide repeated manipulative activities, such as completing value tables and substituting values into simple equations. These activities aim to help students perform interiorization from the Action stage to the Process stage. This is consistent with [29], who explain that interiorization occurs when students repeatedly perform an action until it becomes internalized as a mental process. For medium-ability students who have reached the Process stage, teachers can provide activities for identifying patterns in value increases in tables as a basis for understanding slope before the concept is taught formally.

During the Classroom Discussion stage, teachers facilitate discussions to build the Object and Schema stages. [29] explain that encapsulation from Process to Object occurs when students are able to view a process as a complete object and perform actions on that object. To address the difficulty experienced by medium-ability students who have not achieved encapsulation, discussions should focus on metacognitive questions such as: What is the difference between the slopes of parallel and perpendicular lines? Why is the product of the slopes of perpendicular lines equal to negative one? How are tables, equations, and graphs related? [22] emphasized that errors at the Schema stage constitute the highest category because students do not integrate knowledge systematically. To address the difficulty experienced by high-ability students in drawing graphs, discussion should focus on understanding the Cartesian coordinate system by asking students to explain the rationale for the placement of the axes and to correct common conceptual errors, such as reversing the positions of the axes.

During the Exercises stage, teachers provide graduated practice appropriate to students' ability levels. [20] showed that high-ability students could achieve all APOS stages, medium-ability students could achieve only Action, Process, and Object, and low-ability students could achieve only Action. Therefore, exercises should be differentiated according to ability level. Low-ability students should receive Action-stage exercises, such as determining the slope from equations already written in standard form and completing value tables. Medium-ability students should receive Process-

and Object-stage exercises, such as converting a general equation into standard form, determining the equation of a line through one point with a given slope, and determining the relationship between two lines (parallel or perpendicular). High-ability students should receive Schema-stage exercises, such as contextual word problems incorporating Papuan local wisdom, drawing graphs from given line equations, and analyzing changes in graphs when the slope or constant is changed.

In addition to these three stages, to address the common difficulty experienced by all three subjects in drawing graphs, teachers should provide specific practice in systematic graphing. The practice should include determining the x-intercept by substitution, determining the y-intercept by substitution, correctly plotting the two points on the Cartesian coordinate plane (horizontal and vertical axes), drawing the straight line connecting the two points, and verifying the graph by testing another point in the equation. [25] added that insufficient practice in drawing graphs is one factor contributing to students' errors in solving straight-line equation problems. Through the implementation of the ACE cycle adapted to the specific difficulties at each APOS stage and each student ability category, students' conceptual understanding of straight-line equations is expected to improve gradually, systematically, and continuously. [23] also found that students who were not accustomed to non-routine problems experienced difficulty representing problems graphically. [19] emphasized that the ability to present concepts in various forms of representation, including graphs, is an important indicator of mathematical conceptual understanding.

Limitations of the Study

This study has several limitations: (1) only three subjects participated in the interviews, so the findings cannot be generalized; (2) two students did not participate in the test for technical reasons; and (3) the interview analysis was based only on summaries rather than verbatim transcripts because of time limitations.

4. Conclusion

Based on the findings, this study concludes that mastery of straight-line equations among eighth-grade students at SMP Negeri 5 Jayapura differs according to academic ability level, as examined through the APOS framework. The high-ability student successfully progressed through all four APOS stages, the medium-ability student reached the first three stages, while the low-ability student remained limited to the most basic stage. Interestingly, all three subjects showed weaknesses in graph construction, suggesting cognitive difficulties in forming an integrated Schema from previously understood Objects. Theoretically, this study contributes to the categorization of levels of understanding of straight-line equations through an APOS perspective, while practically it offers input for developing curricula that accommodate Papuan cultural contexts. Future research is recommended to involve a larger number of participants and implement the ACE cycle to bridge students' conceptual understanding gaps in graphical representation.

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